

Complex Analysis - Vvedensky

• The Cauchy Product $\left(\sum_{n=0}^{\infty} a_n\right)\left(\sum_{n=0}^{\infty} b_n\right) = \sum_{n=0}^{\infty} \left(\sum_{k=0}^n a_k b_{n-k}\right)$

•
$$\left. \begin{aligned} e^{i\theta} &= \cos \theta + i \sin \theta \\ e^{-i\theta} &= \cos \theta - i \sin \theta \end{aligned} \right\} \therefore \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}, \quad \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

•
$$\begin{aligned} \cos z &= \cos x \cosh y - i \sin x \sinh y \\ \sin z &= \sin x \cosh y + i \cos x \sinh y \end{aligned}$$

• $\ln z = \ln(re^{i\theta}) = \ln r + \ln e^{i\theta} = \ln r + i\theta$

• $z^a = e^{a \ln z}$

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For $b^2 - 4ac > 0$;

For $b^2 - 4ac = 0$;

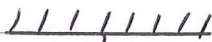
For $b^2 - 4ac < 0$;

$y(x) = Ae^{m_1 x} + Be^{m_2 x}$

$y(x) = (A + Bx)e^{mx}$

$y(x) = Ae^{m_1 x} + Be^{m_2 x}$
 $= e^{\alpha x} (C \cos \beta x + D \sin \beta x)$ \swarrow Im

The Harmonic Oscillator: (About the equilibrium)



"F = ma" (\uparrow):

$$m_0 \frac{d^2 x}{dt^2} = -k(x - \frac{m_0 g}{k}) - r \frac{dx}{dt} - m_0 g$$

$\therefore \ddot{x} + \gamma \dot{x} + \omega_0^2 x = 0$

natural frequency, $\omega_0 = \sqrt{\frac{k}{m_0}}$

damping factor, $\gamma = \frac{r}{m_0}$

$\Rightarrow m = \frac{1}{2}(-\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2})$

For $\gamma^2 - 4\omega_0^2 > 0$; overdamped

For $\gamma^2 - 4\omega_0^2 = 0$; critically damped

For $\gamma^2 - 4\omega_0^2 < 0$; underdamped