1. An electric dipole is formed by two charges $+Q_1$ and $-Q_1$ and situated on the x-axis at $x = -a/2$ and $x = +a/2$ respectively. A second dipole is formed by two further charges of magnitude Q_2, also on the x-axis at $x = X \pm b/2$. The dipoles are widely separated (so that $X \gg a, b$). If the second dipole is in its *stable* (lower energy) alignment with respect to the first, draw a diagram showing:

   (a) the direction of the E field of the first dipole in the vicinity of the second;

   (b) the positions of all four charges;

   (c) the directions of the two dipole moment vectors.

2. For a dipole of dipole moment $\mathbf{p} = p\hat{\mathbf{i}}$ situated at the origin, show that the field on the x-axis can be obtained (in the "far field" approximation) by following these steps.

   Consider the first pair of charges described above (Question 1).

   (a) Write an expression for the exact value of the electric field vector on the x axis at position x using the principle of superposition to obtain

   $$\mathbf{E} = \frac{Q_1}{4\pi\varepsilon_0 x^2}\left[\frac{1}{\left(1+\frac{a}{2x}\right)^2} - \frac{1}{\left(1-\frac{a}{2x}\right)^2}\right];$$

   (b) Use MacLaurin series expansion for the quantity in the square bracket of the form

   $$\frac{1}{(1\pm y)^2} \cong 1 \mp 2y \pm 3y^2 \mp ...$$

   and keep the first two terms only in the expansion;

   (c) introduce $\mathbf{p} = Q_1 a\hat{\mathbf{i}}$ to obtain the final formula:

   $$\mathbf{E} \cong -\frac{\mathbf{p}}{2\pi\varepsilon_0 x^3} \tag{1}$$

   Use the result to show that the potential energy difference between the opposite orientations of dipole 2 in the field of dipole 1 (see Question 1) is

   $$\Delta U \cong \frac{Q_1 Q_2 ab}{\pi\varepsilon_0 X^3}.$$

   [Hint: one way to work out the electrostatic potential energy of the dipole is to calculate the work carried out by moving one of the charges from a position A to its place (say $-a/2$) and then moving the other charge to $+a/2$. The overall potential energy is the sum of the two].

3. Apply a series expansion to Equation (1) to show that the respective electric fields at $x = X \pm b/2$ (i.e. the fields experienced by the charges in the second dipole) are

   $$\mathbf{E} \cong \frac{\mathbf{p}_1}{2\pi\varepsilon_0 X^3}\left(1 \mp \frac{3b}{2X}\right)$$

   where $\mathbf{p}_1$ is the dipole moment of the first dipole.

4. From 3 above how that the non-uniform electric field causes the second dipole to experience a net force of magnitude

   $$F \cong \frac{3p_1 p_2}{2\pi\varepsilon_0 X^4}.$$

   where p_1 and p_2 are the magnitudes of the two dipole moments.

   Is the force attractive or repulsive? Try to understand your conclusion *physically*!

5. The so-called *van der Waals force* between neutral atoms and molecules arises from a "dipole-dipole" interaction. The incessant movement of the bound electrons means that an atom or molecule will exhibit a fluctuating dipole moment p_1, which in turn creates an *induced* dipole moment p_2 in a neighbour.

   Making the reasonable assumption that the magnitude of p_2 is proportional to the field of p_1, convince yourself that the van der Waals force varies as the inverse 7$^{\text{th}}$ power of the separation between them (i.e. force $\sim 1/r^7$ where r is the distance between the molecules).

   Is the van der Waals force attractive or repulsive?

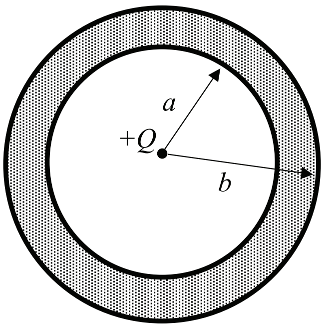
Peter Török

Use the standard convention that $V = 0$ at $r = \infty$ throughout this Classwork.

1. A spherical *insulator* of radius a carries uniform charge density ρ and total charge Q. As shown in the lectures, it is easy to show from Gauss's Flux Law that the electric field *outside* the sphere is given by
$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2} \quad (\text{r} > \text{a}) \tag{1}$$
Show that *inside* the charge distribution ($r < a$), the field is
$$E(r) = \frac{Qr}{4\pi\varepsilon_0 a^3} \quad (\text{r} < \text{a}) \tag{2}$$

2. It was shown in the lectures (both theoretically and experimentally) that charged spherical conducting shells have zero electric field inside. It is an interesting problem then what happens when we put a charge inside without touching the surface of the shell.

   So consider the case of a *conducting* sphere of radius b containing a concentric spherical cavity of radius $a(< b)$, and a point charge $+Q$ at the centre (see diagram). Assuming there is



   no *net* charge on the conductor, find the electric field (a) outside the sphere ($r > b$) (b) within the conductor ($a < r < b$) and (c) in the central cavity ($r < a$).

   What is the total charge on the inner and outer surfaces of the conductor?

3. By integrating Eqs. (1) and (2) in Question 1, show that the corresponding electric potentials are
$$V = \frac{Q}{4\pi\varepsilon_0 r} \quad (\text{r} > \text{a})$$
$$V = \frac{Q}{8\pi\varepsilon_0 a}\left(3 - \frac{r^2}{a^2}\right) \quad (\text{r} < \text{a})$$
Hint: Ensure that V is continuous at $r = a$.

   Draw rough graphs of $E(r)$ and $V(r)$. Is $\partial V/\partial r$ continuous at $r = a$?

4. For the case of Question 2, integrate your expressions for $E(r)$ to obtain corresponding expressions for $V(r)$. Ensure that the potential is continuous at the boundaries. As in Question 3, draw rough graphs of $E(r)$ and $V(r)$. Try to understand the graphs *physically*.

Peter Török

Information needed for this Classwork:

- Capacitance $C = Q/V$
- Stored energy $U = \frac{1}{2}Q^2/C \equiv \frac{1}{2}CV^2$
- $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

Assume that the dielectric constant $\varepsilon_r = 1$ throughout this question.

---

1. (a) The plates of a parallel-plate capacitor (area $8 \times 10^{-3}\text{m}^2$) are separated by a 0.5 mm air gap. Assuming that the dielectric constant of air $\varepsilon_r^{air} \cong 1$, calculate the capacitance.

   If the charge on the plates is ±5 nC, find

   (b) the potential difference V between the plates;

   (c) the electric field E between the plates;

   (d) the stored energy U in the capacitor.

   For a capacitor that is *isolated* (i.e. the charge is constant), consider what happens when the plate separation is doubled. What is the effect on

   (e) the capacitance C;

   (f) the potential difference V;

   (g) the electric field E;

   (h) the stored energy U?

2. The capacitor is now connected to a 9V battery. Find

   (a) the charge $\pm Q$ on the plates;

   (b) the electric field E between the plates;

   (c) the stored energy U.

   If the plate separation is doubled in this case, what is the effect on

   (d) the charge Q;

   (e) the field E;

   (f) the stored energy U?

3. (a) The energy (per unit volume) stored in an electric field is given by $\frac{1}{2}\varepsilon_0\varepsilon_r E^2$. Show that this gives the correct answers for the stored energies in questions 1 and 2.

   (b) For an isolated capacitor (question 1), the force between the plates of the capacitor can be found from the formula
   $$F = -\frac{\partial U}{\partial s}$$
   where s is the plate separation. Obtain an expression for the force. Does it depend on the plate separation?

   (c) In the constant voltage case (question 2), the formula for F is not applicable, at least not without modification. Why is this? Try to find a way to determine the force in this case. Does it depend on the plate separation?

MULTIPLE CHOICE ANSWERS AND THE MAGIC NUMBER

If you add up the Option Numbers for the set of correct answers to questions 1 and 2, you get the "magic number" given at the end. If you get the right number, it doesn't necessarily mean you've got all the answers right, but it does mean that you *might* have done so!

Q	Section	Option	Answer	Q	Section	Option	Answer
1	(a)	1	14.2 pF	2	(a)	1	15.7 pC
		2	142 pF			2	127 pC
		3	17.7 nF			3	1.27 nC
1	(b)	1	28.3 mV	2	(b)	1	18.0 kV/m
		2	17.6 V			2	1.80 kV/m
		3	35.3 V			3	18 V/m
1	(c)	1	70.6 kV/m	2	(c)	1	574 pJ
		2	35.3 kV/m			2	639 pJ
		3	7.09 kV/m			3	5.74 nJ
1	(d)	1	8.85 nJ	2	(d)	1	÷4
		2	88.2 nJ			2	÷2
		3	176 nJ			3	no change
1	(e)	1	÷4			4	×2
		2	÷2			5	×4
		3	no change	2	(e)	1	÷4
		4	×2			2	÷2
		5	×4			3	no change
1	(f)	1	÷4			4	×2
		2	÷2			5	×4
		3	no change	2	(f)	1	÷4
		4	×2			2	÷2
		5	×4			3	no change
1	(g)	1	÷4			4	×2
		2	÷2			5	×4
		3	no change				
		4	×2				
		5	×4				
1	(h)	1	÷4				
		2	÷2				
		3	no change				
		4	×2				
		5	×4				

The magic number is 34

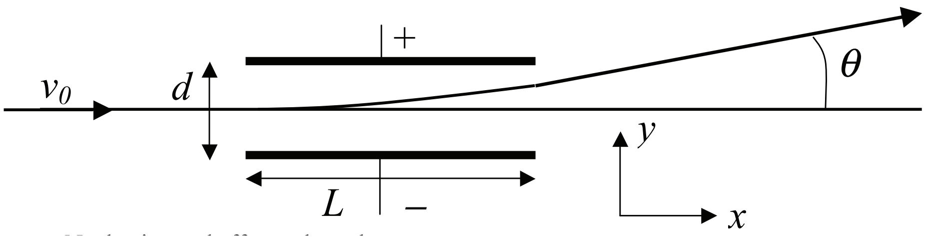
Peter Török

Information needed for this Classwork:

- Lorentz force law: the force on a particle of charge q moving at velocity $\mathbf{v}$ in an electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.
- Velocity of light = 3×10^8 m/s
- elementary charge =1.602×10^{-19} C
- Proton rest mass = 1.67×10^{-27} kg
- electron rest mass = 9.11×10^{-31} kg

The electron was discovered just over 100 years ago by J.J. Thomson working at the Cavendish Laboratory in Cambridge. The details contained in this Classwork are not an exact reflection of Thomson's experiments between 1897 and 1899, but they are close, and the concepts are the same.

1. A beam of electrons moving at a controlled speed pass through a uniform electric field created between two parallel plates of length L, separation d, and potential difference V. As the electrons enter the region between the plates, they are moving at speed v_0 in the x-direction, perpendicular to $\mathbf{E}$ (which is in the -y direction), and as they leave the region, they have an angular deflection θ (see diagram). Neglecting end effects, show that



$$\tan\theta = \frac{VL}{dv_0^2}\left(\frac{e}{m}\right). \qquad (1)$$

2. It appears from the equation derived in Question 1 above that the charge to mass ratio of the electron can be determined in this way. However, in Thomson's "cathode ray" apparatus, the electrons were accelerated to their speed v_0 across a potential difference V_{acc} between the cathode and anode of the cathode ray tube. Show (trivially) that

$$v_0 = \sqrt{\frac{2eV_{acc}}{m}} \qquad (2)$$

and hence that

$$\tan\theta = \frac{V}{V_{acc}}\frac{L}{2d} \qquad (3)$$

which is independent of e/m!

3. Thomson overcame the difficulty by applying a magnetic field to the region between the plates.

    (a) For a magnetic field perpendicular to the plane of the paper, show that the condition for the electron beam to be undeflected is

$$\frac{e}{m} = \frac{E^2}{2V_{acc}B^2} \tag{4}$$

    (b) Should the magnetic field be directed *into* or *out of* the paper?

    (c) Deduce from Eq.(4) that the ratio E/B has the units of velocity.

    (d) If V_{acc}= 300 V, and the measured ratio of E/B is 1.027×10^7 m/s, deduce a value for e/m.

    (e) At what fraction of the velocity of light are the electrons travelling in this case?

    (f) If V = 50 kV and d = 1 cm, what is the value of B?

    (g) If E and B are held steady at these values, what happens (qualitatively) to the track of the electron beam if the value of V_{acc} is increased above 300 V?

4. In the lectures, it was shown that a particle of charge q, moving at speed v in a uniform magnetic field B perpendicular to the plane of the particle's motion, follows a circular path of radius

$$r = \frac{mv}{qB} \tag{5}$$

which it negotiates at a frequency (number of orbits per second)

$$f = \frac{qB}{2\pi m} \tag{6}$$

This is the principle of the device known as the *cyclotron*. Note that f, which is known as the *cyclotron frequency*, is independent of the speed of the particle (at least under non-relativistic conditions), and a function only of B and q/m!

    (a) Find the radius of the track of a 2 MeV proton in a magnetic field of 0.2T.

    (b) Find the cyclotron frequency for electrons in the same magnetic field.

Neglect relativistic effects.

Peter Török

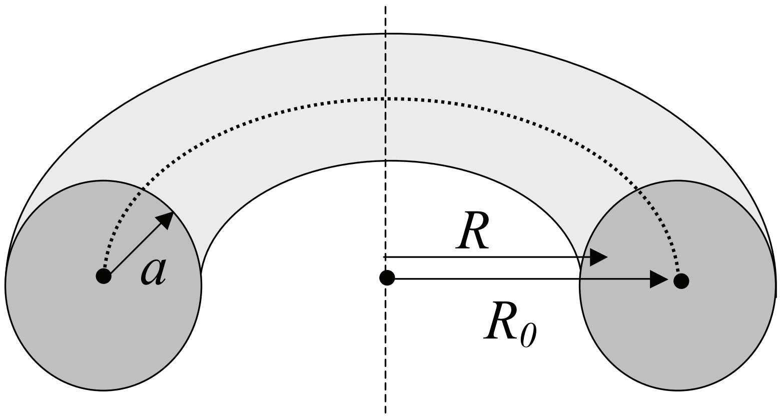
Information needed for this Classwork:
Ampére's law

$$\oint \mathbf{B}.d\mathbf{l} = \mu_0 I$$

The line integral of **B** around any closed path is equal to the current flowing through *any* surface bounded by the path.

---

1. If a uniformly-wound solenoid is bent round into a circle so that its ends meet, you get a toroidal (i.e. doughnut-shaped) solenoid. If the cross section is circular, you can specify the dimensions of the torus in terms of its major and minor radii (R_0 and a respectively; see cut-away diagram).



   In the following questions, R is horizontal distance from the central vertical axis, z is vertical distance above the central horizontal plane, I is the current flowing in the windings (not shown), and N is the number of turns.

   (a) Assuming by symmetry that the magnetic field lines are circles concentric with the central axis, apply Ampère's Law to circular paths to find the strength of the magnetic field both <u>*inside*</u> and <u>*outside*</u> the torus as a function of R and z.

   (b) The internal field is non-uniform. Show that a toroidal solenoid with a major radius R_0 = 3m and a minor radius of a = 1.25m has B_{max}/B_{min} = 2.43. (These are the basic dimensions of the largest machine at JET (= Joint European Torus) which is sited at the Culham Laboratory south of Oxford. However, the cross section is not circular in that case.)

2. The field on the axis of a single circular loop of radius a carrying current I at a distance x from the centre was shown in the lectures to be

$$B_x = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

   Given that $\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2(a^2+x^2)^{1/2}}$, prove that $\int_{-\infty}^{\infty} B_x dx = \mu_0 I$.

Think hard about how to interpret this result in terms of Ampère's Law! Deduce what $\int_{-\infty}^{\infty} B_x dx$ must be for a solenoid of any finite length.

3. **Another problem – for now or later**

A straight piece of wire running along the x-axis between $x = \pm L/2$ forms part of an electrical circuit carrying current I. Its contribution to the magnetic field at a distance R from the origin in the $y - z$ plane can be shown to be

$$B = \frac{\mu_0 IL}{4\pi R(R^2 + L^2/4)^{1/2}}$$

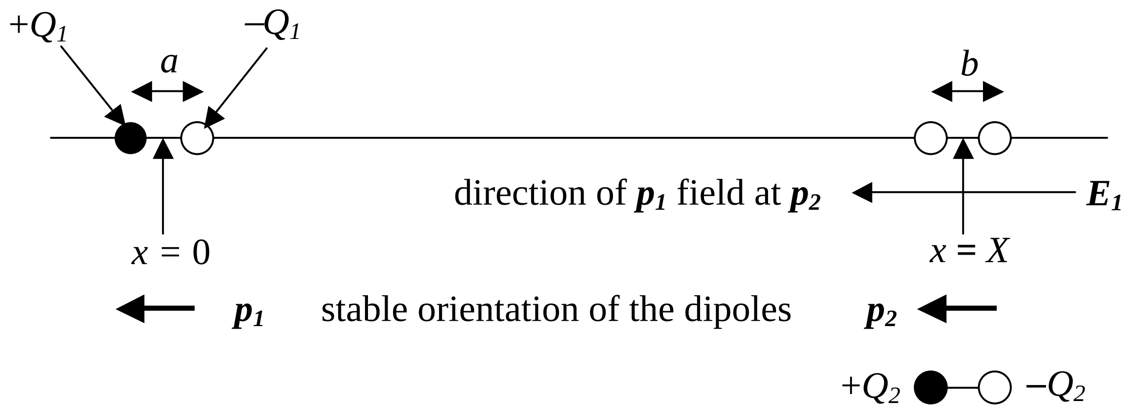
(i) Show that the magnetic field at the centre of a square loop of side L carrying current I is

$$B = \frac{2\sqrt{2}\mu_0 I}{\pi L}$$

(ii) Two identical pieces of wire, both of length Λ are formed into loops, one circular and the other square. If both loops carry the same current, which one produces the stronger magnetic field at its centre, and how much stronger is it?

Peter Török

1.



2. The potential energy of a dipole $\boldsymbol{p}$ in a field $\boldsymbol{E}$ is

$$U = -\boldsymbol{p}.\boldsymbol{E}$$

The difference in potential energy of the opposite orientations is

$$\Delta U = 2pE$$

Since $p = Q_2 b$ and $E \cong \dfrac{Q_1 a}{2\pi\varepsilon_0 X^3}$ it follows that

$$\Delta U \cong \frac{Q_1 Q_2 ab}{\pi\varepsilon_0 X^3}.$$

3. $$\boldsymbol{E}(\text{at } X \pm b/2) = \frac{\boldsymbol{p_1}}{2\pi\varepsilon_0 (X \pm b/2)^3} = \frac{\boldsymbol{p_1}}{2\pi\varepsilon_0 X^3}(1 \pm b/2X)^{-3} \cong \frac{\boldsymbol{p_1}}{2\pi\varepsilon_0 X^3}(1 \mp 3b/2X)$$

Note that both $\boldsymbol{p_1}$ and $\boldsymbol{E_1}$ are both directed in the negative x direction .

4. The net force on dipole 2 in the x-direction is

$$F_x = -\frac{p_1 Q_2}{2\pi\varepsilon_0 X^3}(1 + 3b/2X) - \frac{p_1(-Q_2)}{2\pi\varepsilon_0 X^3}(1 - 3b/2X) = -\frac{3p_1 Q_2 b}{2\pi\varepsilon_0 X^4} = -\frac{3p_1 p_2}{2\pi\varepsilon_0 X^4}$$

Since F_x is negative, the force on dipole 2 is in the negative x direction and is therefore attractive.

5. According to the result of question 3, the field of p_1 varies as X^{-3}. Hence $p_2 \sim X^{-3}$ and (from the result of question 4, $F \sim X^{-7}$.

The van der Waals force is attractive.

1. Applying Gauss's Flux Law to a spherical Gaussian surface of radius r (< a)

$E \times 4\pi r^2 = \dfrac{4\pi r^3 \rho}{3\varepsilon_0}$ which leads to $E = \dfrac{\rho r}{3\varepsilon_0} = \dfrac{Qr}{4\pi\varepsilon_0 a^3}$ since $\dfrac{4\pi a^3 \rho}{3} = Q$.

2. (i) $E(r) = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ $(r > b)$

(ii) $E(r) = 0$ $(a < r < b)$

(iii) $E(r) = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ $(r < a)$

Charge $+Q$ resides on the outer surface and $-Q$ on the inner surface.

3. $V(r) = \dfrac{Q}{4\pi\varepsilon_0 r}$ $(r > a)$

By integrating eq.(2), $V(r) = -\dfrac{Qr^2}{8\pi\varepsilon_0 a^3} + K$, where K must be chosen to ensure continuity of V at $r = a$, i.e. that $V(a) = \dfrac{Q}{4\pi\varepsilon_0 a}$. This yields $K = \dfrac{3Q}{8\pi\varepsilon_0 a}$ and the result follows immediately.

$E(r) = -\dfrac{\partial V}{\partial r}$ is continuous at $r = a$.

4. $V(r) = \dfrac{Q}{4\pi\varepsilon_0 r}$ $(r > b)$

$V(r) = \dfrac{Q}{4\pi\varepsilon_0 b}$ $(a < r < b)$

$V(r) = \dfrac{Q}{4\pi\varepsilon_0 r} + K$ $(r < a)$

where K must be chosen to ensure continuity of V at $r = a$. This leads to $K = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{b} - \dfrac{1}{a}\right)$ and it follows that

$$V(r) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} + \frac{1}{b} - \frac{1}{a}\right)$$

Classwork 3 – ANSWERS

Q	Section	Option	Answer	Q	Section	Option	Answer
1	(a)	1	14.2 Pf	2	(a)	1	15.7 pC
		2*	142 pF			2	127 pC
		3	17.7 nF			3*	1.27 nC
1	(b)	1	28.3 mV	2	(b)	1*	18.0 kV/m
		2	17.6 V			2	1.80 kV/m
		3*	35.3 V			3	18.0 V/m
1	(c)	1*	70.6 kV/m	2	(c)	1	574 pJ
		2	35.3 kV/m			2	639 pJ
		3	7.09 kV/m			3*	5.74 nJ
1	(d)	1	8.85 nJ	2	(d)	1	÷4
		2*	88.2 nJ			2*	÷2
		3	176 nJ			3	no change
1	(e)	1	÷4			4	×2
		2*	÷2			5	×4
		3	no change	2	(e)	1	÷4
		4	×2			2*	÷2
		5	×4			3	no change
1	(f)	1	÷4			4	×2
		2	÷2			5	×4
		3	no change	2	(f)	1	÷4
		4*	×2			2*	÷2
		5	×4			3	no change
1	(g)	1	÷4			4	×2
		2	÷2			5	×4
		3*	no change				
		4	×2				
		5	×4				
1	(h)	1	÷4				
		2	÷2				
		3	no change				
		4*	×2				
		5	×4				
	Score =	21			Score	13	

If you add up the option numbers for the set of correct answers to questions 1 and 2, you get 21 and 13 respectively, as shown. Together these make up the “magic number” of 34 mentioned in the question sheet.

3. (a) This is bound to work because $E = V/s$ and the volume = $A \times s$. Hence (with $\varepsilon_r = 1$), $U = \tfrac{1}{2}\varepsilon_0 E^2 As = \tfrac{1}{2}\varepsilon_0 V^2 A/s = \tfrac{1}{2}CV^2$.

(b) $U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2 s/\varepsilon_0 A$. So $F = -\dfrac{\partial U}{\partial s} = -\tfrac{1}{2}Q^2/\varepsilon_0 A = -Q \times \left(\dfrac{\sigma}{2\varepsilon_0}\right)$. The minus sign implies attraction. The force doesn't depend on the plate separation.

(c) In this case, the work done by the voltage source also needs to included in the energy balance as the plate separation is changed. A simpler and safer procedure is to use the fact that $F = \tfrac{1}{2}QE = \tfrac{1}{2}CV^2/s = \tfrac{1}{2}\varepsilon_0 AV^2/s^2$. This indicates that the force drops by $\times 4$ when *s* is doubled, because both *Q* and *E* are halved.

1st Year Electricity & Magnetism Classwork 4 – ANSWERS

A. Upward force on electron $F_y = eE = eV/d$

Upward acceleration $a_y = eV/md$

Final upward speed $v_y = a_y t = \frac{eV}{md}\frac{L}{v_0}$

where $t = L/v_0$ is the time spent between the plates.

$$\tan\theta = \frac{v_y}{v_x} \equiv \frac{v_y}{v_0} = \frac{eVL}{mdv_0^2} \qquad (1)$$

B. $\frac{1}{2}mv_0^2 = eV_{acc}$ so $v_0 = \sqrt{\frac{2eV_{acc}}{m}}$ (2)

$$\tan\theta = \frac{eVL}{md}\frac{m}{2eV_{acc}} = \frac{LV}{2dV_{acc}} \qquad (3)$$

C. (i) If the electric and magnetic forces are to balance

$$eE = ev_0 B \;\Rightarrow\; \left(\frac{E}{B}\right)^2 = v_0^2 = \frac{2eV_{acc}}{m} \;\Rightarrow\; \frac{e}{m} = \frac{E^2}{2V_{acc}B^2} \qquad (4)$$

(ii) The magnetic forces needs to be in the downward (–*y*) direction, so $\boldsymbol{v}\times\boldsymbol{B}$ needs to be in the +*y* direction, so ***B*** needs to be in the –*z* direction.

(iii) This is obvious from eq.(4).

(iv) $\frac{e}{m} = \frac{(1.027\times10^7)^2}{2\times300} = 1.758\times10^{11}\text{C/kg}$

The ratio e/m is now known to about 9 significant figures. To 6 figures, the value is 1.75882×10^{11}C/kg so the value obtained in this question differs from the exact value rounded to 4 figures by 1 in the last place.

(v) $\frac{v_0}{c} = \frac{1.027\times10^7}{3\times10^8} = 0.034$ (3.4%)

(vi) $\frac{E}{B} = \frac{V/d}{B} = 1.027\times10^7$ m/s, so $B = \frac{5\times10^6}{1.027\times10^7} = 0.487$ T

(vii) If V_{acc} is increased, the magnetic force rises in relation to the electric force. So the beam is deflected downwards (in the –*y* direction).

D. For the proton, $\frac{1}{2}m_p v^2 = 2\times10^6 e$ so $v = \sqrt{4\times10^6\times(e/m_p)}$

$$r = \frac{m_p v}{eB} = \sqrt{\frac{m_p}{e}}\frac{2\times10^3}{0.2} = 1.02\,\text{m}$$

For the electron, $f = \frac{e}{m}\frac{0.2}{2\pi}$=5.60 GHz.

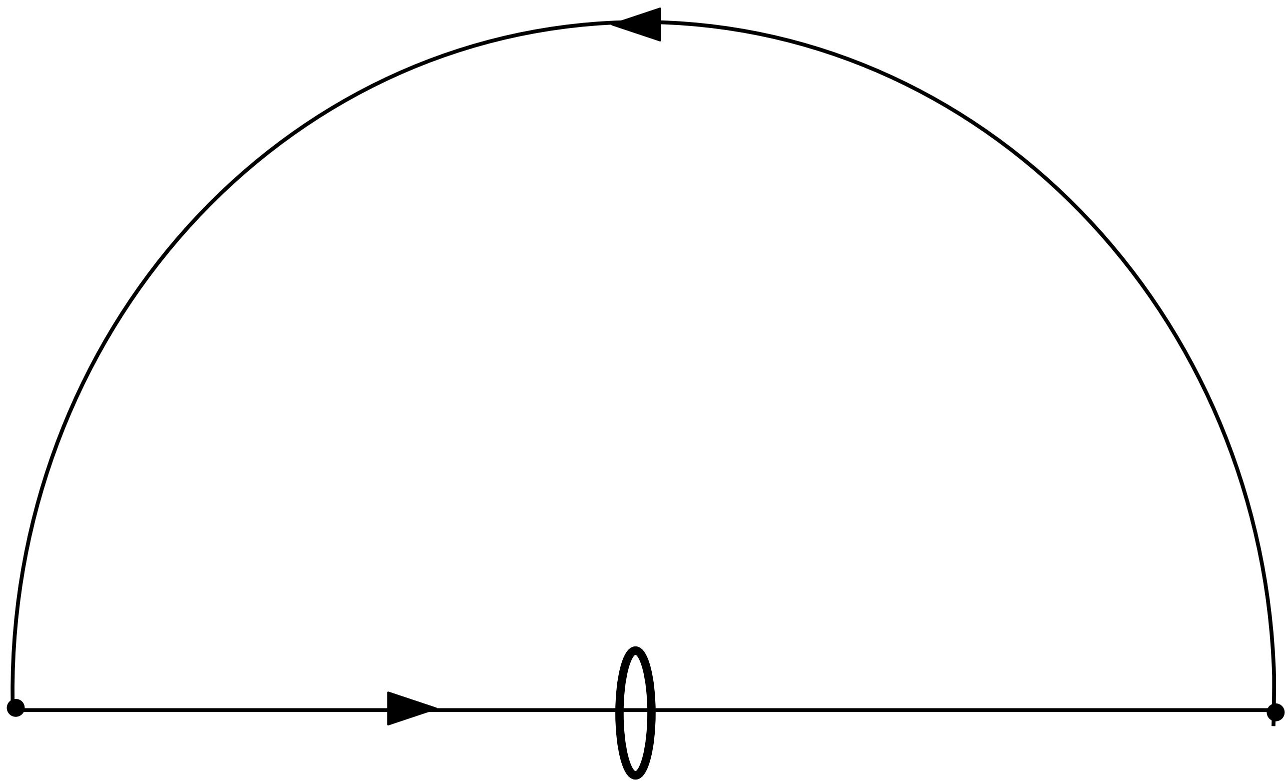
1. (i) Outside the torus, $B = 0$, because circles around the central axis enclose no current.

Inside the torus, $B = \dfrac{\mu_0 NI}{2\pi R}$, so B depends on distance from the central axis.

(ii) $\dfrac{B_{\text{max}}}{B_{\text{min}}} = \dfrac{R_{\text{max}}}{R_{\text{min}}} = \dfrac{R_0 + a}{R_0 - a} = \dfrac{3 + 1.25}{3 - 1.25} = 2.43$

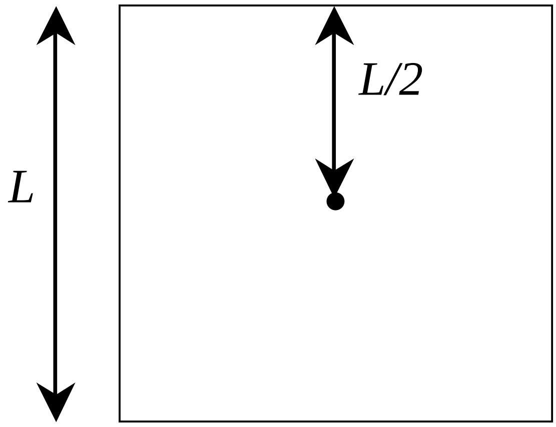
2. $$\frac{\mu_0 I a^2}{2}\int_{-\infty}^{+\infty}\frac{dx}{(a^2+x^2)^{3/2}} = \frac{\mu_0 I a^2}{2}\frac{1}{a^2}\left[\frac{x}{\sqrt{(a^2+x^2)}}\right]_{-\infty}^{+\infty} = \mu_0 I\ .$$

This result makes sense because the line integral along the giant semicircle at infinity needed to complete an "Ampère's Law" circuit around the current loop (see diagram) is zero. This clearly implies that the corresponding line integral along the axis of a finite solenoid will give $\mu_0 I$ also.



3. (i) For a square loop of side L, setting $R = L/2$ yields $B = \dfrac{\mu_0 I\sqrt{2}}{2\pi L}$ from each side.

Thus $B = \dfrac{2\sqrt{2}\mu_0 I}{\pi L}$ for 4 sides.



(ii) Circular loop: $B_c = \dfrac{\mu_0 I}{2a}$ with $\Lambda = 2\pi a$ yields $B_c = \dfrac{\mu_0 I \pi}{\Lambda}$.

Square loop: $B_s = \dfrac{8\sqrt{2}\mu_0 I}{\pi \Lambda}$ since $L = \Lambda / 4$.

Hence $\dfrac{B_c}{B_s} = \dfrac{8\sqrt{2}}{\pi^2} = 1.15$.