The acid test: Lecture 1

1. When you switch on a torch (flashlight) is the amount of current flowing out of the bulb less than, greater than or equal to the current that flows into the bulb? Current at all points around this series circuit must be the same.
2. The supply to a 100 Watt household lamp consists of a 1mm thick copper wire. The wire carries a current of 3A. Find the magnitude of the current density, J in the wire and the drift velocity of the electrons.
   $J = I/A = 3 \div \pi(10^{-3})^2/4 = 3.82\times10^6$ A/m^2. $I = 3 = nqAv_d = 8.5\times10^{28}\times1.6\times10^{-19}\times\div\pi(10^{-3})^2/4\times v_d$ gives $v_d = 0.28$ mm/s
3. Using the values given in question 2 and the value of ρ for copper given earlier calculate the value of the electric field, E, in the wire and the potential difference V between the ends of the wire if it is 1.5 m long.
   $J = 3.82\times10^6$ A/m^2 $= E/\rho$ gives $E = 0.0657$ V/m ($\rho = 1.72\times10^{-8}$), then from $E = V/d$, $V = 0.0657\times1.5 = 0.1$ V.
4. Why are conventional light-bulbs encased in glass and evacuated? Would you expect the resistance of the filament of the light-bulb to increase or decrease when the lamp is switched on?
   The filament is made of metal (usually tungsten) which, when hot, will be susceptible to oxidation unless it is under vacuum. From equation 1.4 and the positive value of α in the table on page 5 the resistance will increase when the filament is hot.
5. An AA type battery is rated at 3000 milliamp-hours (mAh). What is the total charge that an AA battery can deliver during its lifetime?
   The total charge is the maximum current delivered in one hour (3600s)which is $3000\times10^{-3}\times3600 = 10800$ C. Clearly if the battery delivers a much smaller current it will last correspondingly longer.

The acid test: Lecture 2

1. Which is the larger, a gravitational force or an electric force? A way of comparing these is to calculate both for a hydrogen atom: the masses of the electron and proton are 9.11×10^{-31} kg and 1.67×10^{-27} kg and each carries a charge of 1.6×10^{-19} C.

   From equation (2.1) the gravitational force $F=\frac{Gm_{proton}m_{electron}}{r^2}$ where the masses of the proton and electron are given and G = 6.67×10^{-11} $N(m/kg)^2$. Using equation (2.3) and inserting the charge on an electron and proton (both 1.6×10^{-19}C) and $1/4\pi\varepsilon_0 = 8.97\times10^{9}\,Nm^2C^{-2}$ we can calculate the electric force. Since both forces depend on $1/r^2$ which is common to both equations we do not need this value (but it is about 0.1 nm). You should then find that the electric force is much larger than the gravitational force.

2. Figure 2.2 shows two oppositely charged plates. Assuming the separation of the plates to be d, show that the voltage between the plates is related to the electric field by the expression $E=\frac{V}{d}$. (you may assume that the potential at the bottom plate is zero)

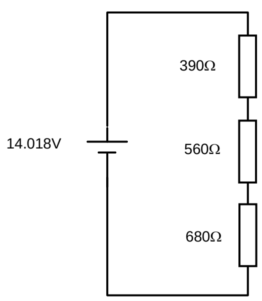
   If we take a positive unit charge q from the lower plate which has a voltage V=0 to the top plate where V=V then we must do work. That work is equal to the force times the distance moved = E×q×d. It is also equal to the change in PE. This difference is equal to $(V_{upper} - V_{lower}) \times q$ where $V_{lower}=0$ and $V_{upper}=V$ giving E=V/d.

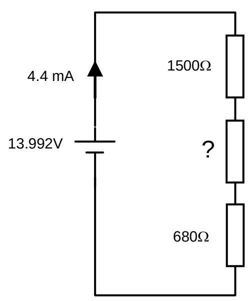
3. For the circuit shown below on the left hand side calculate:

(i) the total resistance 390+560+680 =1630 Ω

(ii) the current through R_1, R_2 and R_3 14.018/1630 = 8.6 mA

(iii) the voltage drop across R_1, R_2 and R_3 V_{R1}= I×R=$8.6\times10^{-3}\times390$ =3.354 V and similarly for V_{R2} and V_{R3}. [4.816 V, 5.848 V]





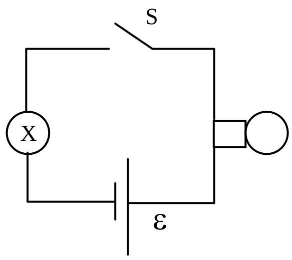
(iv) the total power dissipated in the circuit. P=V×I= $8.6\times10^{-3}\times14.018$=0.12 W

For the circuit shown on the right hand side find the value of the unknown resistor. (1500+680+x)=13.992/0.0044 =1 kΩ.

4. For the circuit shown on the left in question 3, how much energy is given to an electron from the battery to take it around the circuit?

   Energy required equals voltage times charge: $14.018 \times 1.6 \times 10^{-19} = 2.24 \times 10^{-18}$ J

5. A light bulb is connected to a battery of emf ε, a switch S and a component X as shown in the diagram. What is the light output from the bulb if (a) S is open and X is an ammeter? (b) S is open and X is a voltmeter? (c) S is closed and X is an ammeter? (d) S is closed and X is a voltmeter? (a) incomplete circuit no light (b) same (c) ammeters have a negligible resistance and provide no opposition to current flow and the bulb will be bright (d) Voltmeters have extremely large resistances and so the current would be very small, insufficient to make the bulb light up.

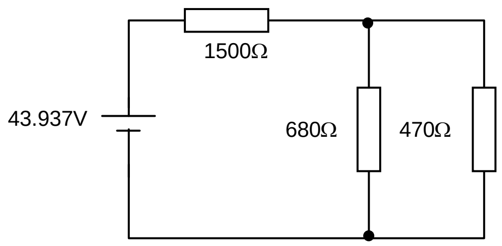


6. The power rating for a car battery is 40 Ah. A car headlamp typically consumes 100W. How long could the battery drive the headlamps if it were not continuously recharged? Car batteries are rated at 12V, so the headlamp voltage is 12V. One headlamp consumes 100W so the current is 100/12= 8.3 A. At a power rating of 40Ah this would last for 40/8.3 = 4.82h but there are two headlamps so the time is reduced to 2.41h. During driving the car engine turns the alternator to continuously recharge the battery.

7. A circuit similar to that shown in figure 2.6(b) consists of a 12 V battery with an internal resistance of 2Ω and a resistor R. What value of R would you need to ensure only a ~1% difference between the battery emf and the measured voltage across R? Voltage divider circuit, battery emf is 12V. Measured voltage across R is $(R/R+2) \times 12$ V. If this is to be within 1% of the battery emf it must equal 11.88V which gives R=198Ω.

The acid test: Lecture 3

1. For the circuit shown right find



(i) the total resistance 1500+ R// where 1/R//=1/680 +1/470 which gives R//= 278Ω so total = 1778 Ω

(ii) the total current
43.937/1778=24.7 mA

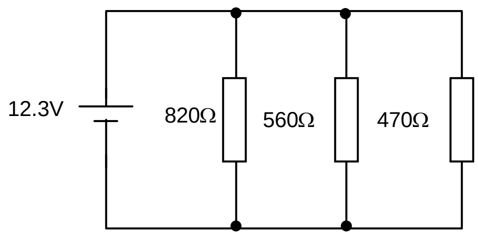
(iii) the current through each resistor
I_{1500}= 24.7 mA. Use current divider rule to show I_{680}= 10.1mA, I_{470}=14.6 mA.

(iv) the voltage across each resistor Use voltage divider rule: V_{1500}= [1500/(1500+278)]×43.937=37.067V. The remainder 43.937-37.067V = 6.87V gives $V_{680,470}$ (same since they are in parallel)

(v) the total power dissipated Power = V×I =43.937×24.7×10^{-3} = 1.085 W

(vi) the power dissipated in each resistor For each resistor use P=V×I to get P_{1500}=915 mW, P_{680}=69.4 mW, P_{470}=100 mW

2. For the circuit shown on the right find



(i) the total resistance Three resistors in parallel $1/R_{total}$ = 1/820+1/560+1/470 =195 Ω

(ii) the total current 12.3/195=63 mA

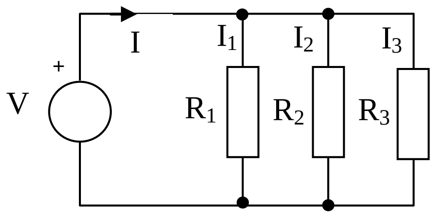
(iii) the current through each resistor The voltage across each resistor is the same (12.3V) so I_{820}=15 mA, I_{560}=21.9 mA, I_{470}=26.2 mA

(iv) the voltage across each resistor 12.3 V

(v) the power dissipated in the 560Ω resistor P=21.9×10^{-3}×12.3=270 mW

3. Show that the total current I in the circuit shown below is given by

$$I = \frac{(R_2R_3 + R_1R_3 + R_1R_2)}{R_1R_2R_3}V$$

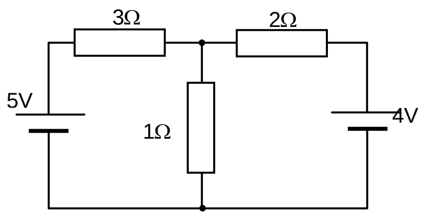


The parallel combination of the three resistors is $1/R_{total}=1/R_1+1/R_2+1/R_3$ which gives $R_{total}=R_1R_2R_3/(R_{12}+R_{23}+R_{13})$ and $I=V/R_{total}$

4. Complete the table:

Gain	V_{out}/V_{in}	dB
1	1	0
10	3.16	10
100	10	20
0.5	0.707	-3
2	1.414	3

5. Show that the current through the 1Ω resistor in figure 3.3 is 2A if the polarity of the 4V battery is reversed (this was a part of a previous examination question and you would have picked up 7 marks for the correct answer).

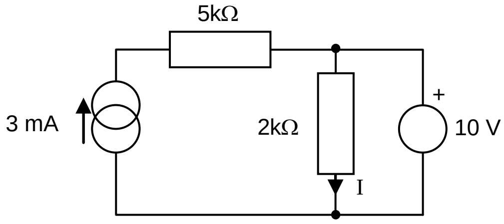


You can do this using Kirchhoff's laws or superposition. Using KVL we obtain two equations, one for each loop which has currents I_1 (left) and I_2 (right):

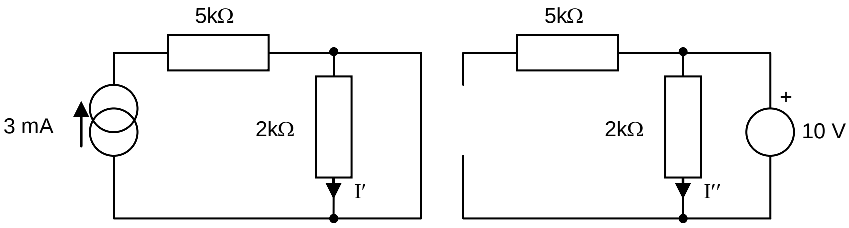
$5 – 3I_1 – 1(I_1+I_2) = 0$ and $4 – 2I_2 – 1(I_1+I_2) = 0$. Solve these two to obtain required result.

The acid test: Lecture 4

1. Find the current I in the circuit shown below:

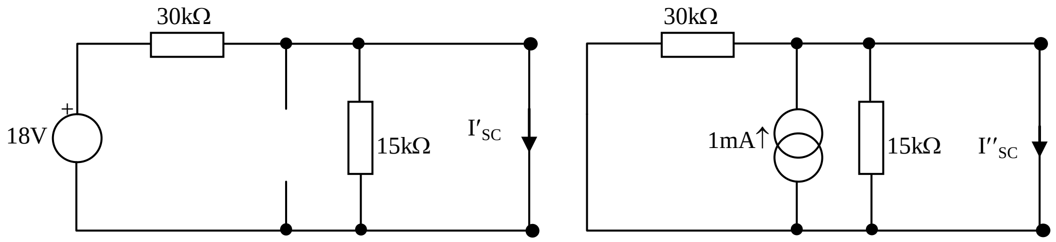


Applying superposition, replace the voltage source by a short circuit (left hand diagram). Then the contribution to I from the current source alone is I′=0 since no current will flow through the 2kΩ resistor (it will take the easy path through the short circuit). Replacing the current source with an open circuit (right hand diagram) the current through the 2kΩ resistor is I′′=10/2000 = 5 mA. So I = 5mA



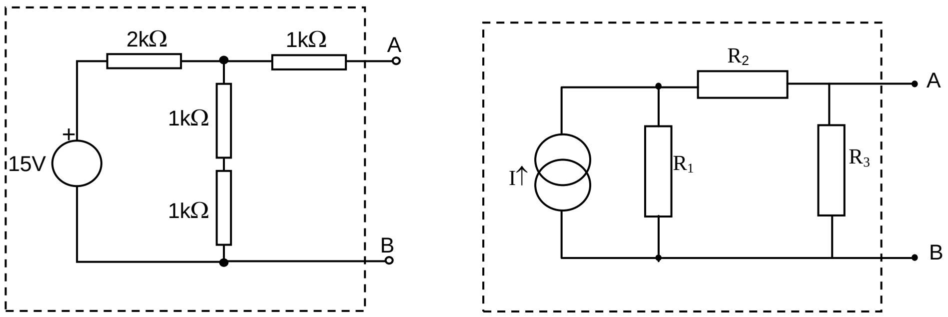
The acid test: Lecture 5

1. Calculate the source resistance of the circuit shown in figure 5.11 using superposition and determine the value of the short circuit current.



The two circuits show the contributions to the short circuit current from the voltage source (left) and current source (right). For I'_{SC} the current will take the easy path and avoid going through the 15kΩ resistor. This gives I'_{SC}=18/30000=0.6mA. For I''_{SC} the current will again take the easy path avoiding both resistors and I''_{SC}=1mA. In both cases the current goes from top to bottom and so the total short circuit current is 1.6mA.

2. Find the Thévenin equivalents of the circuits contained within the dashed lines shown below. A,B are the output terminals.



For the circuit shown on the left we first find V_{OC}: imagine placing an ideal voltmeter across the output terminals AB. No current will pass through the voltmeter (it will have infinite resistance) and consequently no current will pass through the top 1kΩ resistor. Thus V_{OC} is the voltage across the other two 1kΩ resistors which by the voltage divider rule gives [2/(2+2)]×15 = 7.5V. The equivalent circuit can be found by finding I_{SC} or R_S. The latter method is easier – looking into the output terminals (having replaced the voltage source by a short circuit) the resistor network consists of a 1kΩ resistor in series with a parallel combination of a 2kΩ and a (1+1)kΩ resistors which gives 2kΩ. Finding I_{SC} is more time consuming – a wire is placed across AB and the current through the loop which includes the top 1kΩ resistor calculated. R_S is then given by V_{OC}/I_{SC}.

For the circuit shown on the right the solution has already been provided in section 5.3 as $V_{OC} = \frac{R_1 R_3}{(R_1 + R_2 + R_3)} I \quad R_S = \frac{R_3 (R_1 + R_3)}{(R_1 + R_2 + R_3)}$

The acid test: Lecture 6

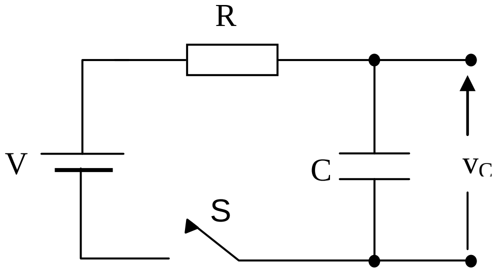
1. Show that the product R and C has units of time.

   R=V/I (Ω or VsC^{-1}) and C=Q/V (CV^{-1}), so RC has units of seconds.

2. A capacitor of value 1 μF is charged to 10 volts. It is then connected in parallel with an uncharged 1 μF capacitor. Calculate the total energy stored (a) before and (b) after the connection is made.

   (a) $U=CV^2/2 = 10^{-6}\times10^2/2 = 50\mu J$ (b) when the two capacitors are connected (but disconnected from the voltage supply) the total charge cannot change (but the voltage across each will) and $Q=CV = 10^{-6}\times10=10^{-5}$ C. However now the total capacitance is 2μF and so $U=Q^2/2C = 10^{-10}/2\times2\times10^{-6} = 25\mu J$

3. When switch S in the circuit shown below is closed the voltage v_C across the



capacitor increases as:

$$v_C = V(1 - e^{-t/RC})$$

where t is the time in seconds after S is closed.

What is the energy stored in the capacitor when it is fully charged?

$$U = CV^2/2$$

By applying Kirchhoff's voltage law to the circuit show that the voltage across the resistor is $v_R = V e^{-t/RC}$

Kirchhoff's law: $V - v_R - v_C = 0$ substitute $v_C = V(1 - e^{-t/RC})$ to obtain $v_R = V e^{-t/RC}$

By considering the power dissipated by the resistor show that the total energy dissipated by the resistor during charging is equal to the total energy delivered to the capacitor.

Power dissipated in the resistor is $P = \int_0^\infty v_R^2 / R \, dt = \frac{V^2}{R}\int_0^\infty e^{-2t/RC} \, dt = \frac{1}{2}CV^2$

Plot the voltages v_R and v_C as a function of time in units of the time constant τ.

Figure 6.5 plots v_C and the current i. Since $v_R = iR$ the two plots can be reproduced in terms of the time constant τ.

The acid test: Lecture 7

1. Calculate the inductance of a coil having 2800 turns, a diameter of 10 cm and a length of 60 cm.

   Equation 7.6 gives $L = \frac{\mu_0 \pi r^2 N^2}{l}$ inserting the appropriate values gives 0.13 H.

2. A 10μH inductor has a diameter of 4mm and a length of 5.7 cm and carries a current of 100mA. What is the energy stored in the magnetic field of the inductor, the magnetic energy density and the magnetic field strength?

   Energy stored is $U = LI^2/2$ which gives $U = 5\times10^{-8}$J. The solenoid volume is $\pi(2\times10^{-2})^2 = 7.16\times10^{-7}$ m^3 so the energy density u_B is $5\times10^{-8}/ 7.16\times10^{-7}$ = $0.07Jm^{-3}$. The magnetic field strength is given by rearranging $\frac{B^2}{2\mu_0}$ given in section 7.4 to give $B = \sqrt{2\mu_0 u_B} = 4.2\times10^{-4}$ T. Note that we didn't require the number of turns on the solenoid (but we did require the length!!)

3. Show that L/R has units of time.

   L has units H or Vs^2/C (see equation (7.5)) and R has units Vs/C.

4. An AC voltage v = 340 cos 100πt is to be converted to a 12V DC voltage. Suggest a suitable ratio of turns on the transformer and a suitable value of capacitor if the "ripple" voltage is to be less than 0.1V (assume the load current is 5 mA).

   Amplitude of AC is 340V, to reduce this to ~12V requires a ratio of primary to secondary turns of 340/12=28. Assume the AC voltage is full wave rectified (see figure (7.5)) then the ripple voltage is given by $\Delta V = \frac{I}{2Cf}$ (section 7.6) where I = 5mA and f, the AC frequency, is 50Hz. If ΔV is ~0.1V then $C=0.005/2\times50\times0.1=0.0005$F or 500μF.

The acid test: Lecture 8

1. An electric kettle is rated at 3 kW, 240V. Calculate the (a) resistance of the element (b) the rms current and the maximum instantaneous power.

   (a) $P=V_{rms} \times I_{rms}$ or V_{rms}^2/R. Using the latter $3000=240^2/R$ so $R=19.2\Omega$ (b) then using the former equation $P=V_{rms} \times I_{rms}$ gives $I_{rms}=12.5A$ while the maximum instantaneous power $P= V\times I = V_{rms}/0.7071 \times I_{rms}/0.7071=$ 6000W.

2. Show that capacitive reactance has units of Ω. $X_C=1/\omega C$ where ω has units of s^{-1} and C has units of C/V. So X_C has units of Vs/C or Ω.

3. A sine-wave voltage $v = 5 \sin \omega t$ is applied across a 1nF capacitor. What is the value of the capacitive reactance and the current flowing to and from the capacitor terminals at (a) 100 Hz; (b) 1 kHz; (c) 10 kHz?

   (a) $X_C=1/\omega C=1/2\pi fC =1/2\pi\times100\times10^{-9}=1.59\times10^{6}\Omega$ (b), (c) use same equation but insert appropriate value of frequency to obtain $1.59\times10^{5}\Omega$, and $1.59\times10^{4}\Omega$. The amplitude of the current is given by $i = v/\omega C$ whilst we must also account for the $\pi/2$ phase lead (CIVIL) of current relative to voltage which gives $i = 3.14\times10^{-6}\sin(628t+\pi/2)$A. A similar approach gives for the other two frequencies: $31.4\times10^{-6}\sin(6283t+\pi/2)$A and $314\times10^{-6}\sin(62832t+\pi/2)$A.

The acid test: Lecture 9

1. A voltage $\mathbf{V}$ = 3 + j2 volts is applied across an impedance $\mathbf{Z}$ = 2 + j5 Ω. Using the complex form of Ohm's law calculate the current flowing in this impedance. Sketch the voltage and current on a phasor diagram and state whether the voltage leads the current or vice versa at this particular frequency.

   Phasor form of Ohm's law: **Z**=**V**/**I** so **I** =**V**/**Z** =(3 + j2)/(2 + j5). Rationalise to obtain **I** = 0.55 – j0.38A Plotting **V** and **I** on an Argand diagram shows that voltage leads current. Alternatively calculate the phase angle tan ϕ = b/a where a is the real part of the complex number and b is the imaginary part for **V** and **I** to confirm your answer.

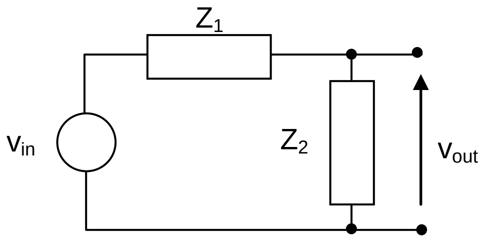
2. Find values for the output voltage v_{out} for the circuit shown below if the input voltage v_{in} is 6 cos 31415t volts:

   (i) If Z_1 and Z_2 are resistances of 1 kΩ and 10 kΩ [5.45cos 31415t V]

   (ii) If Z_1 is a 1kΩ resistance and Z_2 a 30 nF capacitor

   [4.24 cos (31415t + π/4)

   (iii) If Z_1 and Z_2 are capacitors of value 10 nF and 15 nF How will this vary with frequency? [2.46 cos31415t, no frequency change]



   This is an impedance divider circuit so $v_{out} = Z_2/(Z_1+Z_2) \times v_{in}$ (i) If Z_1=1kΩ and Z_2=10kΩ then v_{out}=10/11× 6 cos 31415t = 5.45cos 31415t V. There is no phase change (ii) If Z_1=1kΩ and Z_2 is a capacitor with a reactance Z_2 =1/ωC then v_{out}=1/ωC/(1000+1/ωC)×6 cos 31415t = 5.45cos 31415t

3. Using expressions for the impedance of inductors and capacitors derive the rule for a series combination of two capacitors C_1 and C_2 and a parallel combination of two inductors L_1 and L_2.

For a series combination of capacitors: $Z_{total} = 1/Z_1 + 1/Z_2$ and substitute $Z_{total} = 1/j\omega C_{total}$, $Z_1 = 1/j\omega C_1$, $Z_2 = 1/j\omega C_2$ to give $C_{total} = C_1 + C_2$. Similarly for a parallel combination of inductors $Z_{total} = Z_1 + Z_2$ where $Z_{total} = j\omega L_{total}$, $Z_1 = j\omega L_1$, $Z_2 = j\omega L_2$ to give $L_{total} = L_1 + L_2$.

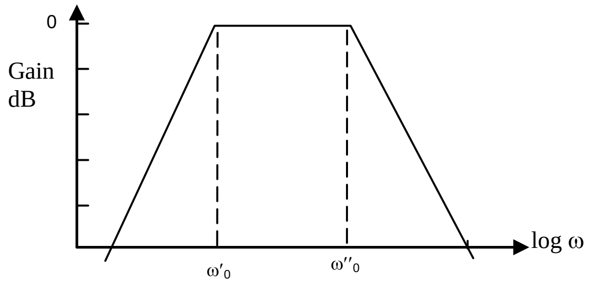
The acid test: Lecture 10

1. A 1 MHz sine wave is input to an amplifier that has a voltage gain of 100. The output is connected to a second amplifier which has a gain of 50, by 2 metres of cable with an attenuation of 1.5 dB per metre. Find the overall gain in dB.

   First amplifier has a gain of 100 (40dB) the second a gain of 50 (34 dB) but there is loss in the cable of 2×1.5 = 3dB. The overall gain is then 40 + 34 - 3 = 71dB.

2. A low pass filter consists of a resistor R_1 = 1500Ω and a capacitor C = 1μF connected to a high pass filter with R_1 = 1500Ω and a capacitor C = 10 nF. Draw the circuit and make a labelled sketch of the Bode plot of the gain (ignore any loading effects of the second circuit on the first).

   First calculate the values of the corner frequency for each circuit (i) LP filter ω'_0=1/CR = 666.7 rad/s (ii) HP filter ω''_0=1/CR = 66666.7 rad/s. The resultant filter looks like:



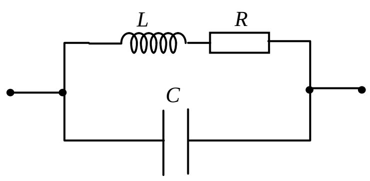
This suggests that the filter would provide zero gain ($v_{out} = v_{in}$) over the frequency range ω''_0 - ω'_0. For frequencies outside this range the gain would reduce at -6dB per octave (-20dB per decade). In practice the two circuits could be joined by a unity gain buffer (covered in lecture 12) which would avoid the loading effects alluded to in the question.

The acid test: Lecture 11

1. A series tuned circuit consists of the following components: an inductor L = 1mH, a capacitor C = 1nF and a resistor R = 10Ω. Calculate the resonant frequency, the bandwidth and the Q factor for this circuit.

   Resonant frequency $\omega_0^2 = 1/LC$ so $\omega_0 = 1/\sqrt{(10^{-3}\times 10^{-9})} = 10^6$ rad/s. Bandwidth = R/L = $10/10^{-3} = 10^4$ rad/s. Q factor $=1/R\times\sqrt{(L/C)} = 0.1\times 10^3 = 100$.

2. Show that the resonant frequency of the circuit shown below is $\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$



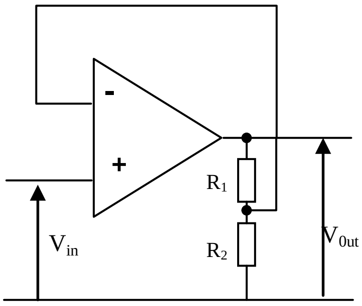
   Method: Find the complex impedance of the series L, R in parallel with the capacitor C. $Z_{LR} = R + j\omega L$ while $1/Z_{total} = 1/Z_{LR} + j\omega C$. The condition for resonance is that Z_{total} (or for that matter $1/Z_{total}$) be real. Substituting for Z_{LR} and rationalising the denominator gives the condition that $1/Z_{total}$ is real: $\omega CR^2 - \omega L + \omega^3 L^2 C = 0$ which yields the given answer.

The acid test: Lecture 12

1. An op-amp has a low frequency gain of 10^5. DC voltages of 2mV and 1mV are applied to the inverting and non-inverting inputs. What is the output voltage v_{out}?

   $V_{out} = 10^5(1 - 2)\times10^{-3}$ = -100 mV

2. What is the closed loop gain and the bandwidth of the feedback amplifier shown in circuit below if the open loop gain is $10^4/(1 + jf/10)$, R_1=95kΩ and R_2=5kΩ? Make a labelled sketch of v_{out} if v_{in} is (i) 5×10^{-3} cos 100t (ii) 5×10^{-3} cos 31410t. [20, 100 mV, 70 mV]



In this configuration the closed loop gain G is $(R_1+R_2)/R_2$ = 20, using the values given. The values of the gain bandwidth product is derived from the open loop gain equation as $10^4\times10=10^5$. Now using the closed loop gain in the gain bandwidth product gives $10^5 = 20\times f'$ where f′ is the new corner frequency or bandwidth. Its value is $10^5/20$= 5000Hz.

(i) for an input of 5×10^{-3} cos 100t the frequency is $100/2\pi$ = 16Hz. This is well below f′ and the output will be 100×10^{-3} cos 100t

(ii) for an input of 5×10^{-3} cos 31410t the frequency is $31410/2\pi$ = 5000Hz which equals f′ the corner frequency where we know the gain has dropped by -3dB or $0.7071\times100\times10^{-3}$ cos 31410t. Thus the amplitude will be ~70 mV and there will be a phase lag of $\pi/4$ compared with v_{in}.