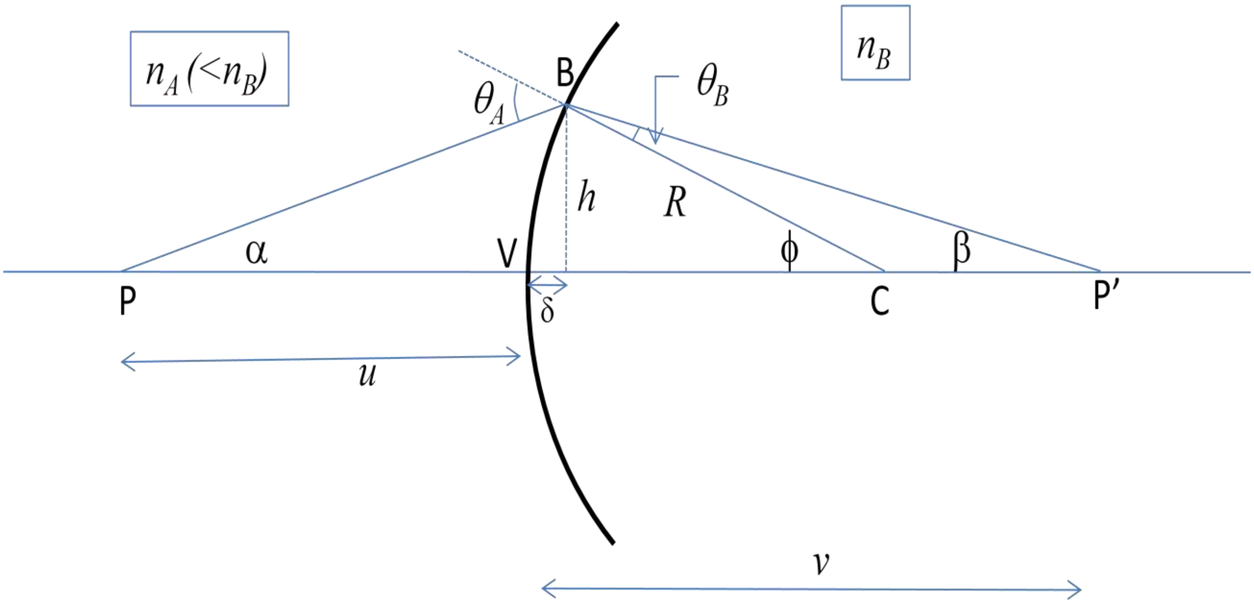
2^{nd} Year Optics 2011-12
Classwork 1 – Refraction at a single curved surface

The purpose of this classwork is to take some of the ideas that we explored with a mirror and apply them to the imaging properties of a single curved refracting surface. A similar approach will also be taken when we consider thin lenses in lectures.

Here we are considering the situation we looked at in the last lecture:



where we derived the following equation:

$$\frac{n_A}{u} + \frac{n_B}{v} = (n_B - n_A)\frac{1}{R}$$

This equation relates the object position u, and image position v, with the refractive indices of the media and the radius of curvature of the refracting surface.

1. Find expressions for the focal lengths (note the plural) of the refractive surface. If medium B is glass with refractive index 1.5, medium A is air (n=1) and the radius of curvature is 100cm, what are the values of the two focal lengths?
2. Draw a ray diagram showing how the image of an extended object of lateral extent y is formed. To do this you will need to identify the main ray tracing rules of this situation. There are four rules which you should identify, but in your diagram it is easiest to consider the two cases of a ray which passes though the centre of curvature, and a ray which is incident on the curved surface at the vertex V.
3. To determine the lateral magnification identify the two triangles in your ray diagram which both have a vertex at V, and include the object height and distance, and the image height and distance. Unlike when a mirror was considered, these triangles are not similar as they do not share the same angle. How are the two angles at the vertex related? (This should have been one of your ray-tracing rules.)

   Using the small angle approximation, find an expression for the lateral magnification.
4. If you have time, draw ray diagrams (as in 2) for the cases where the surface is concave (negative R) and where $n_A > n_B$. What form does the image take in each case – virtual or real?

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Classwork 2 – Interference and Diffraction

The purpose of this classwork is to give some familiarity with the principles of interference and diffraction and a feel for the characteristic values of some of the quantities involved.

1. Consider a Young's slits apparatus with the following parameters:

   Distance from light source to S_0, from S_0 to $S_{1,2}$ and from $S_{1,2}$ to screen all = 0.5m
   Width of source slit S_0 = 0.1 mm
   Separation of S_1 and S_2 = 0.5 mm
   Wavelength = 600 nm

   a) What is the spacing of the fringes on the screen?
   b) What is the width of the first diffraction peak of S_0 at the double slit? Is it sufficient to illuminate both slits coherently?
   c) Compare your calculation in (b) to the condition given in lectures for the interference pattern not getting washed out. Do they give the same result?
   d) If the width of each of the double slits is 0.05 mm, estimate how many interference fringes will be observed (Hint: consider diffraction at the double slit).

2. The *visibility* of fringes is defined by

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

   where I_{max} and I_{min} are the maximum and minimum intensities respectively. Calculate the visibility for the following ratio of amplitudes of E_1 and E_2:

   a) $E_1 / E_2 = 1$
   b) $E_1 / E_2 = 0.5$
   c) $E_1 / E_2 = 0.1$

   This shows that it is still relatively easy to observe fringes even when one source is significantly weaker than the other.

3. The principal diffraction peaks produced by a diffraction grating occur at angles given by the equation

$$d \sin\theta = m\lambda$$

   where m is an integer, λ is the wavelength and *d* is the spacing between the lines on the grating. For a diffraction grating with 600 lines per mm and an overall width of 3 cm, calculate the following:

   a) At what angle does the first diffraction peak occur for a wavelength of 633 nm?
   b) At what angle does the second diffraction peak for this wavelength occur?
   c) Why is there no third order diffraction peak?
   d) Find the angular width of each of these peaks (use formulae from the lecture).

1. $$\frac{n_A}{u}+\frac{n_B}{v}=(n_B-n_A)\frac{1}{R}$$

To find focal lengths take *u* and *v*, in turn, to equal infinity.

$u=\infty$

$$\frac{n_A}{\infty}+\frac{n_B}{v}=(n_B-n_A)\frac{1}{R} \qquad v=f_B=\frac{n_B}{(n_B-n_A)}R$$

Where f_B is the focal length in medium B

$v=\infty$

$$\frac{n_A}{u}+\frac{n_B}{\infty}=(n_B-n_A)\frac{1}{R} \qquad u=f_A=\frac{n_A}{(n_B-n_A)}R$$

Where f_A is the focal length in medium A.

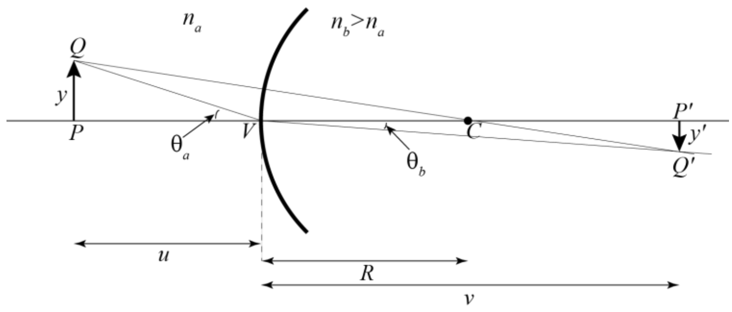
With n_B=1.5 and n_A=1, f_A =2R, f_B =3R, and R=100cm gives f_A =200cm, f_B =300cm.

2.

Ray Tracing rules:

1. A ray parallel to the axis will pass through the focal point in the second medium.
2. A ray passing through or appearing to come from the focal point travels parallel to the axis in the second medium.
3. A ray passing through the centre of curvature of the surface meets the surface at normal incidence and is not refracted.
4. A ray incident at the vertex is refracted according to Snell's law (as the optic axis is a normal).

Note the first two of these need a to be drawn carefully, and to apply all four, you need to know refractive indices. For the purpose of getting to a general expression for the magnification it is easier to use only the last two.



3.

For lateral magnification consider the triangle PQV, P'Q'V which give us

$$\tan(\theta_A) = \frac{y}{u}, \qquad \tan(\theta_B) = -\frac{y'}{v}$$

(y' is negative as it is below the optic axis) We know that the two angles are related by Snell's law:

$$n_A \sin(\theta_A) = n_B \sin(\theta_B)$$

In the small angle approximation *tan(θ)=sin(θ)*, hence we have

$$n_A \frac{y}{u} = -n_B \frac{y'}{v}$$

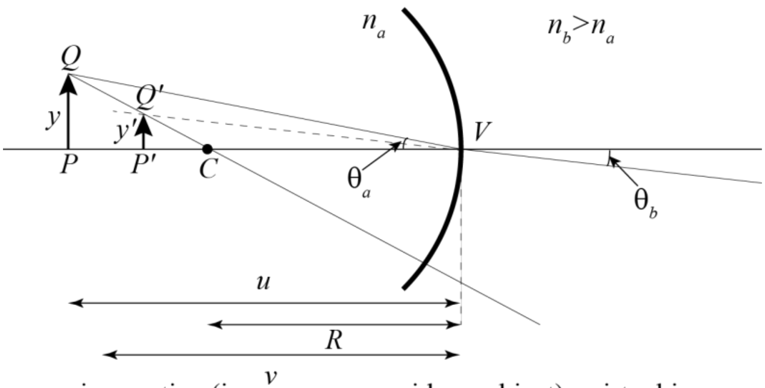
and the magnification:

$$M = \frac{y'}{y} = -\frac{n_A}{n_B}\frac{v}{u}$$

(negative meaning the image is inverted).

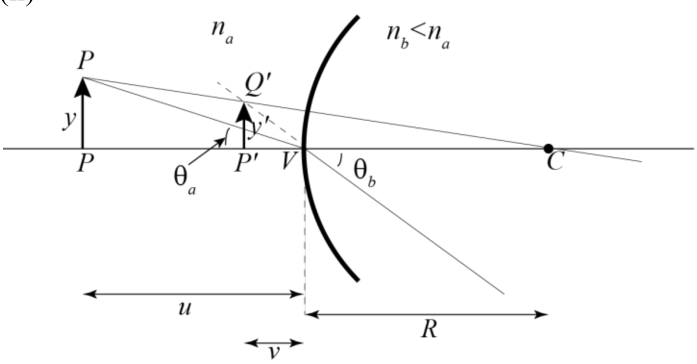
4.

(i)



v is negative (image on same side as object) - virtual image

(ii)



v is negative (image on same side as object) - virtual image

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Classwork 2 – Solutions

1. Distance, D, from light source to S_0, from S_0 to $S_{1,2}$ and from $S_{1,2}$ to screen all = 0.5m
   Width of source slit S_0 a = 0.1 mm
   Separation of S_1 and S_2 d= 0.5 mm
   Wavelength λ= 600 nm
   a) Angular spacing $\theta = \frac{\lambda}{d}$, spacing on screen $x = R\frac{\lambda}{d} = 500\frac{0.6}{500} = 0.6mm$
   b) Width of the first diffraction peak of S_0 $2\frac{\lambda}{a} \times D = 6mm$. Separation of slits is 0.5mm so easily illuminated.
   c) Lecture calculation gave $a < \frac{\lambda D}{d}$ Same to within factor of 2.
   d) Width of diffraction pattern from 0.05 mm slit is $2\frac{\lambda}{a} \times D = 12mm$, separation of fringes 0.6mm so 20 fringes within central peak..

2. The *visibility* of fringes is defined by

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + i_{\min}}, \qquad I \propto |E|^2$$

   a) $E_1 / E_2 = 1$, $I_{\max} = (1+1)^2 = 4,\ I_{\min} = (1-1)^2 = 0,\quad V = 1$
   b) $E_1 / E_2 = 0.5$ $I_{\max} = (1+0.5)^2 = \frac{9}{4},\ I_{\min} = (1-0.5)^2 = \frac{1}{4},\quad V = 0.8$
   c) $E_1 / E_2 = 0.1$ $I_{\max} = (1+0.1)^2 = 1.21,\ I_{\min} = (1-0.1)^2 = 0.91,\quad V = 0.2$

   Visibility of 0.2 can easily be seen.

3. The principal diffraction peaks produced by a diffraction grating occur at angles given by the equation

$$d\sin\theta = m\lambda$$

   where *m* is an integer, λ is the wavelength and *d* is the spacing between the lines on the grating. Diffraction grating has 600 lines per mm and an overall width of 3 cm. d (=1/600) =1.67μm.:

   a) $d\sin\theta = \lambda, \Rightarrow \sin\theta = \frac{\lambda}{d} = \frac{0.633}{1.67} \Rightarrow \theta = 22.3^0$
   b) $d\sin\theta = 2\lambda, \Rightarrow \sin\theta = \frac{\lambda}{d} = \frac{2\times 0.633}{1.67} \Rightarrow \theta = 49.3^0$
   c) $d\sin\theta = 3\lambda, \Rightarrow \sin\theta = \frac{\lambda}{d} = \frac{3\times 0.633}{1.67} > 1$, so no third order.
   d) $\delta\theta = \frac{\lambda}{D\cos\theta},\ \delta\theta(m=1) = 2.3\times 10^{-5},\ \delta\theta(m=1) = 3.2\times 10^{-5}$, note they are not the same.