$[J_z, J^2] = 0$

$[J_\pm, J^2] = 0$

$[J_z, J_\pm] = \pm\hbar J_\pm$ with $J_\pm \equiv J_x \pm i J_y$

$[J_i, J_j] = i\hbar \varepsilon_{ijk} J_k$

$\dim \mathcal{H}_j = 2j+1$

$\langle u|A|v\rangle^* = \langle v|A^\dagger|u\rangle$

$A|a_n\rangle = \lambda_n|a_n\rangle \Rightarrow \langle a_n|A^\dagger = \lambda_n^* \langle a_n|$

$N = a^\dagger a$

$N|n\rangle = n|n\rangle$

$$\begin{cases} a|n\rangle = \sqrt{n}\,|n-1\rangle \\ a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle \end{cases} \Rightarrow \quad a|0\rangle = 0, \quad |n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle$$

$\langle n|a^\dagger a|n\rangle = \langle n-1|n-1\rangle B_n^2$

$a^\dagger a|n\rangle = n|n\rangle$

and

$\langle n|a a^\dagger|n\rangle = \langle n+1|n+1\rangle C_n^2$

$a a^\dagger|n\rangle = (a^\dagger a + [a, a^\dagger])|n\rangle = (n+1)|n\rangle$

$$f(A) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} A^k$$

$$\rho = \sum_n p_n |\psi_n\rangle\langle\psi_n|$$

$$\int dp\, e^{\frac{ip}{\hbar}(x-y)} = 2\pi\hbar\,\delta(x-y)$$